Analytical Solutions to Eqs.1 & 2

$$\Big\{\Big\{\mathrm{Tat}_D(t) \to$$

$$\Big(e^{-\frac{1}{2}t\left(\delta - k_{p300} + k_{SIRT1} + \sqrt{k_{p300}^2 + (-2\delta + 6 k_{SIRT1} + 4 k_{TR})k_{p300} + (\delta - k_{SIRT1})^2}\right)}\Big) \Big(-e^{t\sqrt{k_{p300}^2 + (-2\delta + 6 k_{SIRT1} + 4 k_{TR})k_{p300} + (\delta - k_{SIRT1})^2}}$$

$$\delta c_1 + \delta c_1 + \Big(-1 + e^{t\sqrt{k_{p300}^2 + (-2\delta + 6 k_{SIRT1} + 4 k_{TR})k_{p300} + (\delta - k_{SIRT1})^2}}\Big)k_{p300}\, c_1 +$$

$$e^{t\sqrt{k_{p300}^2 + (-2\delta + 6 k_{SIRT1} + 4 k_{TR})k_{p300} + (\delta - k_{SIRT1})^2}}\sqrt{(k_{p300}^2 + (-2\delta + 6 k_{SIRT1} + 4 k_{TR})k_{p300} +}$$

$$(\delta - k_{SIRT1})^2)\, c_1 + \sqrt{(k_{p300}^2 + (-2\delta + 6 k_{SIRT1} + 4 k_{TR})k_{p300} + (\delta - k_{SIRT1})^2)}\, c_1 +$$

$$\Big(-1 + e^{t\sqrt{k_{p300}^2 + (-2\delta + 6 k_{SIRT1} + 4 k_{TR})k_{p300} + (\delta - k_{SIRT1})^2}}\Big)(c_1 + 2 c_2)\, k_{SIRT1} +$$

$$2 e^{t\sqrt{k_{p300}^2 + (-2\delta + 6 k_{SIRT1} + 4 k_{TR})k_{p300} + (\delta - k_{SIRT1})^2}}\, c_2\, k_{TR} - 2 c_2\, k_{TR}\Big)\Big/$$

$$(2\sqrt{(k_{p300}^2 + (-2\delta + 6 k_{SIRT1} + 4 k_{TR})k_{p300} + (\delta - k_{SIRT1})^2)}),$$

$$\mathrm{Tat}_A(t) \to \Big(e^{-\frac{1}{2}t\left(\delta - k_{p300} + k_{SIRT1} + \sqrt{k_{p300}^2 + (-2\delta + 6 k_{SIRT1} + 4 k_{TR})k_{p300} + (\delta - k_{SIRT1})^2}\right)}$$

$$\Big(\Big(-1 + e^{t\sqrt{k_{p300}^2 + (-2\delta + 6 k_{SIRT1} + 4 k_{TR})k_{p300} + (\delta - k_{SIRT1})^2}}\Big)(2 c_1 - c_2)\, k_{p300} +$$

$$c_2\Big(\sqrt{(k_{p300}^2 + (-2\delta + 6 k_{SIRT1} + 4 k_{TR})k_{p300} + (\delta - k_{SIRT1})^2)}$$

$$\Big(1 + e^{t\sqrt{k_{p300}^2 + (-2\delta + 6 k_{SIRT1} + 4 k_{TR})k_{p300} + (\delta - k_{SIRT1})^2}}\Big) +$$

$$\Big(-1 + e^{t\sqrt{k_{p300}^2 + (-2\delta + 6 k_{SIRT1} + 4 k_{TR})k_{p300} + (\delta - k_{SIRT1})^2}}\Big)\delta -$$

$$\Big(-1 + e^{t\sqrt{k_{p300}^2 + (-2\delta + 6 k_{SIRT1} + 4 k_{TR})k_{p300} + (\delta - k_{SIRT1})^2}}\Big)k_{SIRT1}\Big)\Big)\Big)\Big/$$

$$(2\sqrt{(k_{p300}^2 + (-2\delta + 6 k_{SIRT1} + 4 k_{TR})k_{p300} + (\delta - k_{SIRT1})^2)})\Big\}\Big\}$$

When an equation for GFP :

$$\frac{d}{dt} GFP = IRES \times k_{TR} \times Tat_A - \gamma\, GFP$$

is included in the system of equations the solutions for
$Tat_D(t)$ and $Tat_A(t)$ do not change and the solution for $GFP(t)$ is :

$$GFP(t) \rightarrow \left(e^{-\frac{1}{2} t\left(\delta - k_{p300} + k_{SIRT1} + \sqrt{k_{p300}^2 + (-2\delta + 6 k_{SIRT1} + 4 k_{TR})\, k_{p300} + (\delta - k_{SIRT1})^2}\right)} \right.$$

$$\left(\left(2\, e^{\frac{1}{2} t\left(-2\gamma + \delta - k_{p300} + k_{SIRT1} + \sqrt{k_{p300}^2 + (-2\delta + 6 k_{SIRT1} + 4 k_{TR})\, k_{p300} + (\delta - k_{SIRT1})^2}\right)}\right)\gamma\, c_3 - $$

$$4\, e^{\frac{1}{2} t\left(-2\gamma + \delta - k_{p300} + k_{SIRT1} + \sqrt{k_{p300}^2 + (-2\delta + 6 k_{SIRT1} + 4 k_{TR})\, k_{p300} + (\delta - k_{SIRT1})^2}\right)} k_{SIRT1}\, c_3 + $$

$$\left(-\left(1 + e^{t\sqrt{k_{p300}^2 + (-2\delta + 6 k_{SIRT1} + 4 k_{TR})\, k_{p300} + (\delta - k_{SIRT1})^2}}\right.\right. - $$

$$\left.2\, e^{\frac{1}{2} t\left(-2\gamma + \delta - k_{p300} + k_{SIRT1} + \sqrt{k_{p300}^2 + (-2\delta + 6 k_{SIRT1} + 4 k_{TR})\, k_{p300} + (\delta - k_{SIRT1})^2}\right)}\right)\!IRES$$

$$(c_1 - c_2) - 2\, e^{\frac{1}{2} t\left(-2\gamma + \delta - k_{p300} + k_{SIRT1} + \sqrt{k_{p300}^2 + (-2\delta + 6 k_{SIRT1} + 4 k_{TR})\, k_{p300} + (\delta - k_{SIRT1})^2}\right)}$$

$$\left.c_3\right) k_{TR}\right) k_{p300}^3 + $$

$$\left(-24\, e^{\frac{1}{2} t\left(-2\gamma + \delta - k_{p300} + k_{SIRT1} + \sqrt{k_{p300}^2 + (-2\delta + 6 k_{SIRT1} + 4 k_{TR})\, k_{p300} + (\delta - k_{SIRT1})^2}\right)} c_3\, k_{SIRT1}^2 + \right.$$

$$2\left(5\, e^{\frac{1}{2} t\left(-2\gamma + \delta - k_{p300} + k_{SIRT1} + \sqrt{k_{p300}^2 + (-2\delta + 6 k_{SIRT1} + 4 k_{TR})\, k_{p300} + (\delta - k_{SIRT1})^2}\right)}\right)(\gamma + \delta)\, c_3 + $$

$$\left(-3\left(1 + e^{t\sqrt{k_{p300}^2 + (-2\delta + 6 k_{SIRT1} + 4 k_{TR})\, k_{p300} + (\delta - k_{SIRT1})^2}}\right.\right. - $$

$$\left.\left.2\, e^{\frac{1}{2} t\left(-2\gamma + \delta - k_{p300} + k_{SIRT1} + \sqrt{k_{p300}^2 + (-2\delta + 6 k_{SIRT1} + 4 k_{TR})\, k_{p300} + (\delta - k_{SIRT1})^2}\right)}\right)\right)$$

$$IRES\,(c_1 - c_2) - 14$$

$$\left.e^{\frac{1}{2} t\left(-2\gamma + \delta - k_{p300} + k_{SIRT1} + \sqrt{k_{p300}^2 + (-2\delta + 6 k_{SIRT1} + 4 k_{TR})\, k_{p300} + (\delta - k_{SIRT1})^2}\right)} c_3\right) k_{TR}\right)$$

$$k_{SIRT1} - 4\left(\left(1 + e^{t\sqrt{k_{p300}^2 + (-2\delta + 6 k_{SIRT1} + 4 k_{TR})\, k_{p300} + (\delta - k_{SIRT1})^2}} - 2\right.\right.$$

$$\left.e^{\frac{1}{2} t\left(-2\gamma + \delta - k_{p300} + k_{SIRT1} + \sqrt{k_{p300}^2 + (-2\delta + 6 k_{SIRT1} + 4 k_{TR})\, k_{p300} + (\delta - k_{SIRT1})^2}\right)}\right)\!IRES$$

$$\left.(c_1 - c_2) + 2\, e^{\frac{1}{2} t\left(-2\gamma + \delta - k_{p300} + k_{SIRT1} + \sqrt{k_{p300}^2 + (-2\delta + 6 k_{SIRT1} + 4 k_{TR})\, k_{p300} + (\delta - k_{SIRT1})^2}\right)} c_3\right)$$

$$k_{TR}^2 + 2\, e^{\frac{1}{2} t\left(-2\gamma + \delta - k_{p300} + k_{SIRT1} + \sqrt{k_{p300}^2 + (-2\delta + 6 k_{SIRT1} + 4 k_{TR})\, k_{p300} + (\delta - k_{SIRT1})^2}\right)} \gamma\,(\gamma - 3\delta)\, c_3 + $$

$$k_{TR}\left(4\, e^{\frac{1}{2} t\left(-2\gamma + \delta - k_{p300} + k_{SIRT1} + \sqrt{k_{p300}^2 + (-2\delta + 6 k_{SIRT1} + 4 k_{TR})\, k_{p300} + (\delta - k_{SIRT1})^2}\right)}\right)(2\gamma + \delta)\, c_3 + $$

$$\mathbf{IRES}\left(\left(\left(1 + e^{t\sqrt{k_{p300}^2 + (-2\delta + 6k_{SIRT1} + 4k_{TR})\,k_{p300} + (\delta - k_{SIRT1})^2}}\right. \right.\right.$$

$$\left. - 2\,e^{\frac{1}{2}t\left(-2\gamma + \delta - k_{p300} + k_{SIRT1} + \sqrt{k_{p300}^2 + (-2\delta + 6k_{SIRT1} + 4k_{TR})\,k_{p300} + (\delta - k_{SIRT1})^2}\right)}\right)\delta$$

$$(2c_1 - 3c_2) + \left(1 + e^{t\sqrt{k_{p300}^2 + (-2\delta + 6k_{SIRT1} + 4k_{TR})\,k_{p300} + (\delta - k_{SIRT1})^2}}\right.$$

$$\left. - 2\,e^{\frac{1}{2}t\left(-2\gamma + \delta - k_{p300} + k_{SIRT1} + \sqrt{k_{p300}^2 + (-2\delta + 6k_{SIRT1} + 4k_{TR})\,k_{p300} + (\delta - k_{SIRT1})^2}\right)}\right)\gamma$$

$$c_2 + \left(-1 + e^{t\sqrt{k_{p300}^2 + (-2\delta + 6k_{SIRT1} + 4k_{TR})\,k_{p300} + (\delta - k_{SIRT1})^2}}\right)(c_1 - c_2)$$

$$\left.\left.\left.\sqrt{(k_{p300}^2 + (-2\delta + 6k_{SIRT1} + 4k_{TR})\,k_{p300} + (\delta - k_{SIRT1})^2)}\right)\right)\right)k_{p300}^2 -$$

$$\left(4\,e^{\frac{1}{2}t\left(-2\gamma + \delta - k_{p300} + k_{SIRT1} + \sqrt{k_{p300}^2 + (-2\delta + 6k_{SIRT1} + 4k_{TR})\,k_{p300} + (\delta - k_{SIRT1})^2}\right)}c_3\,k_{SIRT1}^3 + \right.$$

$$\left(10\,e^{\frac{1}{2}t\left(-2\gamma + \delta - k_{p300} + k_{SIRT1} + \sqrt{k_{p300}^2 + (-2\delta + 6k_{SIRT1} + 4k_{TR})\,k_{p300} + (\delta - k_{SIRT1})^2}\right)}(\gamma - 2\delta)c_3 + \right.$$

$$\left(\left(1 + e^{t\sqrt{k_{p300}^2 + (-2\delta + 6k_{SIRT1} + 4k_{TR})\,k_{p300} + (\delta - k_{SIRT1})^2}}\right.\right.$$

$$\left. - 2\,e^{\frac{1}{2}t\left(-2\gamma + \delta - k_{p300} + k_{SIRT1} + \sqrt{k_{p300}^2 + (-2\delta + 6k_{SIRT1} + 4k_{TR})\,k_{p300} + (\delta - k_{SIRT1})^2}\right)}\right)\mathbf{IRES}$$

$$(c_1 - c_2) + 2\,e^{\frac{1}{2}t\left(-2\gamma + \delta - k_{p300} + k_{SIRT1} + \sqrt{k_{p300}^2 + (-2\delta + 6k_{SIRT1} + 4k_{TR})\,k_{p300} + (\delta - k_{SIRT1})^2}\right)}$$

$$\left.c_3\right)k_{TR}\right)k_{SIRT1}^2 +$$

$$\left(k_{TR}\left(4\,e^{\frac{1}{2}t\left(-2\gamma + \delta - k_{p300} + k_{SIRT1} + \sqrt{k_{p300}^2 + (-2\delta + 6k_{SIRT1} + 4k_{TR})\,k_{p300} + (\delta - k_{SIRT1})^2}\right)}\right.\right.$$

$$(2\gamma - 3\delta)c_3 + \mathbf{IRES}\left(-2\left(1 + e^{t\sqrt{k_{p300}^2 + (-2\delta + 6k_{SIRT1} + 4k_{TR})\,k_{p300} + (\delta - k_{SIRT1})^2}}\right.\right.$$

$$\left. - 2\,e^{\frac{1}{2}t\left(-2\gamma + \delta - k_{p300} + k_{SIRT1} + \sqrt{k_{p300}^2 + (-2\delta + 6k_{SIRT1} + 4k_{TR})\,k_{p300} + (\delta - k_{SIRT1})^2}\right)}\right)$$

$$\delta(c_1 - 4c_2) - 6\left(1 + e^{t\sqrt{k_{p300}^2 + (-2\delta + 6k_{SIRT1} + 4k_{TR})\,k_{p300} + (\delta - k_{SIRT1})^2}}\right.$$

$$\left. - 2\,e^{\frac{1}{2}t\left(-2\gamma + \delta - k_{p300} + k_{SIRT1} + \sqrt{k_{p300}^2 + (-2\delta + 6k_{SIRT1} + 4k_{TR})\,k_{p300} + (\delta - k_{SIRT1})^2}\right)}\right)$$

$$\gamma\,c_2 + \left(-1 + e^{t\sqrt{k_{p300}^2 + (-2\delta + 6k_{SIRT1} + 4k_{TR})\,k_{p300} + (\delta - k_{SIRT1})^2}}\right)(c_1 + 3c_2)$$

$$\left.\left.\sqrt{(k_{p300}^2 + (-2\delta + 6k_{SIRT1} + 4k_{TR})\,k_{p300} + (\delta - k_{SIRT1})^2)}\right)\right) -$$

$$4\,e^{\frac{1}{2}t\left(-2\gamma + \delta - k_{p300} + k_{SIRT1} + \sqrt{k_{p300}^2 + (-2\delta + 6k_{SIRT1} + 4k_{TR})\,k_{p300} + (\delta - k_{SIRT1})^2}\right)}$$

$$\left.(3\gamma^2 - 3\delta\gamma - 2\delta^2)c_3\right)k_{SIRT1} +$$

$$2\,e^{\frac{1}{2}t\left(-2\gamma + \delta - k_{p300} + k_{SIRT1} + \sqrt{k_{p300}^2 + (-2\delta + 6k_{SIRT1} + 4k_{TR})\,k_{p300} + (\delta - k_{SIRT1})^2}\right)}\gamma(2\gamma - 3\delta)\delta\,c_3 +$$

$$2\,\mathbf{IRES}\,c_2\,k_{TR}^2\left(\sqrt{(k_{p300}^2 + (-2\delta + 6k_{SIRT1} + 4k_{TR})\,k_{p300} + (\delta - k_{SIRT1})^2)}\right.$$

$$\left(-1 + e^{t\sqrt{k_{p300}^2 + (-2\delta + 6k_{SIRT1} + 4k_{TR})\,k_{p300} + (\delta - k_{SIRT1})^2}}\right) -$$

$$2 \left(1 + e^{t \sqrt{k_{p300}^2 + (-2\delta + 6 k_{SIRT1} + 4 k_{TR}) k_{p300} + (\delta - k_{SIRT1})^2}} - 2 \right.$$

$$\left. e^{\frac{1}{2} t \left(-2\gamma + \delta - k_{p300} + k_{SIRT1} + \sqrt{k_{p300}^2 + (-2\delta + 6 k_{SIRT1} + 4 k_{TR}) k_{p300} + (\delta - k_{SIRT1})^2}\right)}\right) \gamma +$$

$$2 \left(1 + e^{t \sqrt{k_{p300}^2 + (-2\delta + 6 k_{SIRT1} + 4 k_{TR}) k_{p300} + (\delta - k_{SIRT1})^2}} - 2 \right.$$

$$\left. e^{\frac{1}{2} t \left(-2\gamma + \delta - k_{p300} + k_{SIRT1} + \sqrt{k_{p300}^2 + (-2\delta + 6 k_{SIRT1} + 4 k_{TR}) k_{p300} + (\delta - k_{SIRT1})^2}\right)}\right) \delta \right) +$$

$$k_{TR} \left(\text{IRES} \left(\left(1 + e^{t \sqrt{k_{p300}^2 + (-2\delta + 6 k_{SIRT1} + 4 k_{TR}) k_{p300} + (\delta - k_{SIRT1})^2}} - \right.\right.\right.$$

$$2 e^{\frac{1}{2} t \left(-2\gamma + \delta - k_{p300} + k_{SIRT1} + \sqrt{k_{p300}^2 + (-2\delta + 6 k_{SIRT1} + 4 k_{TR}) k_{p300} + (\delta - k_{SIRT1})^2}\right)}\right)$$

$$(c_1 - 3 c_2) \delta^2 + \left(\left(-1 + e^{t \sqrt{k_{p300}^2 + (-2\delta + 6 k_{SIRT1} + 4 k_{TR}) k_{p300} + (\delta - k_{SIRT1})^2}}\right) \right.$$

$$\sqrt{(k_{p300}^2 + (-2\delta + 6 k_{SIRT1} + 4 k_{TR}) k_{p300} + (\delta - k_{SIRT1})^2)}$$

$$(c_1 - 2 c_2) + 2 \left(1 + e^{t \sqrt{k_{p300}^2 + (-2\delta + 6 k_{SIRT1} + 4 k_{TR}) k_{p300} + (\delta - k_{SIRT1})^2}} - \right.$$

$$\left. 2 e^{\frac{1}{2} t \left(-2\gamma + \delta - k_{p300} + k_{SIRT1} + \sqrt{k_{p300}^2 + (-2\delta + 6 k_{SIRT1} + 4 k_{TR}) k_{p300} + (\delta - k_{SIRT1})^2}\right)}\right)$$

$$\gamma c_2\Big) \delta - \left(-1 + e^{t \sqrt{k_{p300}^2 + (-2\delta + 6 k_{SIRT1} + 4 k_{TR}) k_{p300} + (\delta - k_{SIRT1})^2}}\right) \gamma$$

$$(2 c_1 - c_2) \sqrt{(k_{p300}^2 + (-2\delta + 6 k_{SIRT1} + 4 k_{TR}) k_{p300} + (\delta - k_{SIRT1})^2)}\Big) -$$

$$2 e^{\frac{1}{2} t \left(-2\gamma + \delta - k_{p300} + k_{SIRT1} + \sqrt{k_{p300}^2 + (-2\delta + 6 k_{SIRT1} + 4 k_{TR}) k_{p300} + (\delta - k_{SIRT1})^2}\right)}$$

$$(4\gamma^2 - 4\delta\gamma - \delta^2) c_3\Big)\Big) k_{p300} + (\gamma - \delta)(\delta - k_{SIRT1})$$

$$\left(2 e^{\frac{1}{2} t \left(-2\gamma + \delta - k_{p300} + k_{SIRT1} + \sqrt{k_{p300}^2 + (-2\delta + 6 k_{SIRT1} + 4 k_{TR}) k_{p300} + (\delta - k_{SIRT1})^2}\right)} c_3 k_{SIRT1}^2 - \right.$$

$$\left(2 e^{\frac{1}{2} t \left(-2\gamma + \delta - k_{p300} + k_{SIRT1} + \sqrt{k_{p300}^2 + (-2\delta + 6 k_{SIRT1} + 4 k_{TR}) k_{p300} + (\delta - k_{SIRT1})^2}\right)} (\gamma + \delta) c_3 + \right.$$

$$\left(1 + e^{t \sqrt{k_{p300}^2 + (-2\delta + 6 k_{SIRT1} + 4 k_{TR}) k_{p300} + (\delta - k_{SIRT1})^2}} - 2 \right.$$

$$\left. e^{\frac{1}{2} t \left(-2\gamma + \delta - k_{p300} + k_{SIRT1} + \sqrt{k_{p300}^2 + (-2\delta + 6 k_{SIRT1} + 4 k_{TR}) k_{p300} + (\delta - k_{SIRT1})^2}\right)}\right) \text{IRES } c_2$$

$$k_{TR}\Big) k_{SIRT1} + 2 e^{\frac{1}{2} t \left(-2\gamma + \delta - k_{p300} + k_{SIRT1} + \sqrt{k_{p300}^2 + (-2\delta + 6 k_{SIRT1} + 4 k_{TR}) k_{p300} + (\delta - k_{SIRT1})^2}\right)}$$

$$\gamma \delta c_3 + \text{IRES } c_2 k_{TR} \left(\sqrt{(k_{p300}^2 + (-2\delta + 6 k_{SIRT1} + 4 k_{TR}) k_{p300} + (\delta - k_{SIRT1})^2)}\right.$$

$$\left(-1 + e^{t \sqrt{k_{p300}^2 + (-2\delta + 6 k_{SIRT1} + 4 k_{TR}) k_{p300} + (\delta - k_{SIRT1})^2}}\right) +$$

$$\left(1 + e^{t \sqrt{k_{p300}^2 + (-2\delta + 6 k_{SIRT1} + 4 k_{TR}) k_{p300} + (\delta - k_{SIRT1})^2}} - 2 \right.$$

$$\left.\left.\left.\left.\left. e^{\frac{1}{2} t \left(-2\gamma + \delta - k_{p300} + k_{SIRT1} + \sqrt{k_{p300}^2 + (-2\delta + 6 k_{SIRT1} + 4 k_{TR}) k_{p300} + (\delta - k_{SIRT1})^2}\right)}\right) \delta \right)\right)\right)\right) \Big/$$

$$(2 ((\gamma - \delta)(\gamma - k_{SIRT1}) + k_{p300} (\gamma - 2 k_{SIRT1} - k_{TR})))$$

$$(k_{p300}^2 +$$
$$(-2\,\delta + 6\,k_{SIRT1} + 4\,k_{TR})$$
$$k_{p300} + (\delta - k_{SIRT1})^2))$$

For simplicity we can substitute numerical values into the above solutions to obtain the following:

Simplify[gfpsolns //. $\{k_{p300} \to .01, k_{SIRT1} \to 1, k_{TR} \to .1, \delta \to 2, \gamma \to 0.5, IRES \to 10\}$]

$\{\{\text{Tat}_D(t) \to e^{-2.00099\,t}\,((0.989141 + 0.0108591\,e^{1.01198\,t})\,c_1 + (-1.08698 + 1.08698\,e^{1.01198\,t})\,c_2),$
$\quad \text{Tat}_A(t) \to e^{-2.00099\,t}\,((-0.00988164 + 0.00988164\,e^{1.01198\,t})\,c_1 + (0.0108591 + 0.989141\,e^{1.01198\,t})\,c_2),$
$\quad \text{GFP}(t) \to e^{-2.00099\,t}\,((0.00658342 - 0.0202074\,e^{1.01198\,t} + 0.013624\,e^{1.50099\,t})\,c_1 +$
$\qquad (-0.0072346 - 2.02274\,e^{1.01198\,t} + 2.02997\,e^{1.50099\,t})\,c_2 + 1.\,e^{1.50099\,t}\,c_3)\}\}$